Electromagnetic soliton damping in a ferromagnetic medium

V. Veerakumar[1] and M. Daniel[2]
[1]Department of Metallurgical Engineering, Regional Engineering College, Tiruchirapalli 620 015, India
[2]Centre for Nonlinear Dynamics, Department of Physics, Bharathidasan University, Tiruchirapalli 620 024, India
(Received 24 July 1997)

We study the propagation of an electromagnetic wave in an isotropic damped ferromagnetic medium with free charges. When the magnitude of the damping is small the excitations of magnetization of the medium and the magnetic induction and magnetic field of the electromagnetic wave are governed by solitons. When the damping increases the electromagnetic solitons decelerate and the shape distorts at one end. Also the electromagnetic soliton gets damped as time progresses. [S1063-651X(97)06412-X]

PACS number(s): 41.20.−q, 11.10.Lm, 75.60.Ej

I. INTRODUCTION

The propagation of an electromagnetic wave (EMW) in ordered magnetic media especially in a ferromagnetic medium has become very useful in the context of technologically important magneto-optical recordings for higher storage and fast reading [1]. In a recent paper Daniel, Veerakumar, and Amuda [2] found that in a charge-free ferromagnetic medium the propagation of an EMW introduces an effective field equivalent to the single ion uniaxial anisotropy and constant external magnetic field. It was also observed that the magnetic induction lies in a plane normal to the direction of propagation. Further, the excitation of the magnetization of the ferromagnetic medium is restricted to the normal plane at the lowest order of perturbation and goes out of the plane at higher orders. Also, the excitations of the magnetization and magnetic induction and hence the magnetic field were found to be governed by soliton modes. It is understood that the study of EMW propagation in ferromagnetic media with free charges and current density is very important in ferrite devices such as ferrite loaded waveguides at microwave frequencies [3,4]. Recently, Nakata [5] and also Leblond [6] separately investigated the nonlinear modulation of an EMW during propagation in a ferrite medium with free charges and Gilbert damping. However, both descriptions do not take into account the effect of the current density created by the moving charges, which is essential in the case of a medium with damping. Motivated by this, in the present paper we investigate the propagation of an EMW in an isotropic ferromagnetic medium by taking into account the effect of damping and the presence of current density together. In Sec. II we formulate the model and derive the dynamical equations to be solved. In Sec. III the magnetization dynamics and the EMW propagation is treated using a perturbation theory when the magnitude of damping is small. The effect of damping on the excitations of magnetization, magnetic induction, and hence on the magnetic field of the EMW when it is high is investigated in Sec. IV. Finally, the results are concluded in Sec. V.

II. MODEL AND DYNAMICAL EQUATIONS

The Maxwell equations of electromagnetics [7] are written as

$$\nabla\cdot\mathbf{E}=(1/\epsilon_0)\,\rho, \tag{1a}$$

$$\nabla\cdot\mathbf{B}=0, \tag{1b}$$

$$\nabla\wedge\mathbf{E}=-(\partial\mathbf{B}/\partial t), \tag{1c}$$

$$\nabla\wedge\mathbf{H}=\mathbf{J}+\epsilon_0(\partial\mathbf{E}/\partial t). \tag{1d}$$

Here the fields $\mathbf{H}=(H^x,H^y,H^z)$ and $\mathbf{E}=(E^x,E^y,E^z)$ have the usual meaning of the magnetic and electric fields, respectively. $\mathbf{B}=(B^x,B^y,B^z)$ is the magnetic induction and ϵ_0 is the dielectric constant of the medium. ρ is the free charge density and $\mathbf{J}=(J^x,J^y,J^z)$ is the current density given by $\mathbf{J}=\sigma[\mathbf{E}+\mathbf{V}\wedge\mathbf{B}]$, where σ is the conductivity of the medium and $\mathbf{V}$ is the velocity with which the charges are moving. For slowly moving charges the current density given above takes the form $\mathbf{J}=\sigma\mathbf{E}$; using this and Eq. (1a) in the continuity equation for the free current given by $\nabla\cdot\mathbf{J}=-\partial\rho/\partial t$, we obtain $\rho(t)=\rho(0)\exp(-\sigma/\epsilon_0)t$. This shows that any initial free charge density $\rho(0)$ dissipates, i.e., it flows out to the edges in a characterestic time $\tau=\epsilon_0/\sigma$. Therefore, we can consider $\rho=0$ and hence Eq. (1a) takes the form $\nabla\cdot\mathbf{E}=0$. In the case of untreated ferromagnetic materials, the magnetization, the magnetic induction, and the magnetic field are connected by the linear relation [7]

$$\mathbf{H}=(\mathbf{B}/\mu_0)-\mathbf{M}, \tag{2}$$

where μ_0 is the permeability of the material. Now, taking the curl on both sides of Eq. (1d) and using Eqs. (1b), (1c), and (2) and the relation $\mathbf{J}=\sigma\mathbf{E}$, after a little algebra we obtain

$$c^2\nabla^2\mathbf{B}-\frac{\partial^2\mathbf{B}}{\partial t^2}-\frac{\sigma}{\epsilon_0}\frac{\partial\mathbf{B}}{\partial t}=\frac{1}{\epsilon_0}[\nabla^2\mathbf{M}-\nabla(\nabla\cdot\mathbf{M})]. \tag{3}$$

Here $c=1/\sqrt{\mu_0\epsilon_0}$ is the velocity of the propagation of the EMW in the medium. Equation (3) describes the evolution of the magnetic induction component of the EMW when it propagates in a ferromagnetic medium with magnetization $\mathbf{M}(\mathbf{r},t)$.

The evolution of the magnetization $\mathbf{M}(\mathbf{r},t)$ in an isotropic ferromagnetic medium with Gilbert damping in the presence of an external magnetic field $\mathbf{H}(\mathbf{r},t)$ (in the classical continuum limit) can be expressed in terms of the Landau-Lifshitz equation [8]

$$\frac{\partial\mathbf{M}}{\partial t}=\mathbf{M}\wedge[\nabla^2\mathbf{M}+2A\mathbf{H}]+\alpha\{\mathbf{M}\wedge[\mathbf{M}\wedge(\nabla^2\mathbf{M}+2A\mathbf{H})]\}. \tag{4}$$

The first term on the right-hand side of Eq. (4) represents the contribution due to the exchange interaction, the term proportional to A corresponds to the Zeeman energy, and the term proportional to α represents the Gilbert damping due to the relativistic interaction. Conventionally α is identified as the dimensionless Gilbert damping parameter that can be measured by means of a linewidth in a standard ferromagnetic resonance absorption experiment [3,9]. α can be treated as a constant value as its variations with frequency are of higher order. In Eq. (4) $2A=g\mu_B$, where g is the gyromagnetic ratio and μ_B is the Bohr magneton. Equation (4) in one dimension has already been studied independently as an interesting nonlinear dynamical model of the classical continuum Heisenberg ferromagnetic spin chain exhibiting magnetic soliton excitations when $\alpha=0$ [10] and soliton damping when $\alpha\neq 0$ [11,12]. Now the set of coupled equations (3) and (4) completely describes the propagation of an EMW in an isotropic ferromagnetic medium with Gilbert damping in the presence of free charges.

III. ELECTROMAGNETIC SOLITON WHEN DAMPING IS SMALL

In order to find the nature of the propagation of the EMW in the ferromagnetic medium with free charges and damping, we now try to solve the coupled dynamical equations (3) and (4) after making a perturbation analysis. It was recently found [2] that when the conductivity σ of the medium and the damping parameter α are negligibly small (i.e., when $\sigma=\alpha=0$) the excitations of the magnetization of the medium and the magnetic induction and magnetic field of the EMW are governed by solitons. Now in order to investigate the effect of damping we introduce the wave variable $\hat{\xi}=x-vt$ and stretch the wave variable and time variable by introducing

$$\xi=\varepsilon\hat{\xi}, \tag{5a}$$

$$\tau=\varepsilon^3 t, \tag{5b}$$

where ϵ is a very small parameter. Here the velocity v may be identified [13,14] as $v=[\lambda/(1+\lambda)]^{1/2}c$, where $\lambda=\mathbf{H}_0/\mathbf{M}_0$. $\mathbf{H}_0,\mathbf{M}_0$ are the values of the magnetic field and magnetization in the unperturbed uniform state. As the effects of conductivity and damping are small compared to the exchange interaction and the external field, we rescale the conductivity σ and the damping parameter α by using the small parameter ε as

$$\sigma=\varepsilon^3\sigma, \tag{6a}$$

$$\alpha=\varepsilon^2\alpha. \tag{6b}$$

Moreover, we have chosen the above specific rescaling because the conductivity is small compared to the damping since the damping includes the effects due to the exchange interaction and external field. To study the weak nonlinear and damping effects we then expand the magnetization $\mathbf{M}$ and the magnetic induction $\mathbf{B}$ using the small parameter ε as

$$\mathbf{S}^i=\sum_{j=0}\varepsilon^j\mathbf{S}_j^i, \quad i=1,2, \tag{7}$$

where $\mathbf{S}^1$'s and $\mathbf{S}^2$'s represent $\mathbf{M}$ and $\mathbf{B}$, respectively. We now substitute Eqs. (6) and (7) in the one-dimensional (say, x) component equations (3) and (4), collect the coefficients of different powers of ε, and solve the resultant equations. For example, on solving the equations at $O(\varepsilon^0)$, we obtain

$$B_0^x=0, \tag{8a}$$

$$B_0^y=k^{-1}M_0^y, \tag{8b}$$

$$B_0^z=k^{-1}M_0^z, \tag{8c}$$

$$M_0^x=0, \tag{8d}$$

where $k=\epsilon_0(c^2-v^2)$. Similarly, on solving the equations at $O(\varepsilon^1)$, we obtain

$$B_1^x=0, \tag{9a}$$

$$B_1^y=k^{-1}M_1^y, \tag{9b}$$

$$B_1^z=k^{-1}M_1^z, \tag{9c}$$

and

$$M_1^x=(\mu_0 v/2AB_0^z)\ (\partial M_0^y/\partial\xi) \tag{9d}$$

or

$$M_1^x=(\mu_0 v/2AB_0^y)\ (\partial M_0^z/\partial\xi). \tag{9e}$$

Finally, at $O(\varepsilon^2)$ after using Eqs. (8) we obtain

$$B_2^x=0, \tag{10a}$$

$$\frac{\partial}{\partial\xi}[kB_2^y-M_2^y]=-v\left[2\epsilon_0\frac{\partial B_0^y}{\partial\tau}-\sigma B_0^y\right], \tag{10b}$$

$$\frac{\partial}{\partial\xi}[kB_2^z-M_2^z]=-v\left[2\epsilon_0\frac{\partial B_0^z}{\partial\tau}-\sigma B_0^z\right]. \tag{10c}$$

and

$$\begin{aligned}&v\frac{\partial M_1^x}{\partial\xi}+[M_0^y(\partial^2M_0^z/\partial\xi^2)-M_0^z(\partial^2M_0^y/\partial\xi^2)]\\&+\frac{2A}{\mu_0}\{[M_0^yB_2^z-M_2^zB_0^y+M_1^yB_1^z-M_1^zB_1^y+M_2^yB_0^z-M_0^zB_2^y]\\&+\alpha[B_0^x(M_0^{x^2}-1)+M_0^x(M_0^yB_0^y+M_0^zB_0^z)]\}=0.\end{aligned} \tag{10d}$$

Similarly, the equations for the y and z components of $\mathbf{M}_1$ can be obtained by replacing the components of $\mathbf{M}$ and $\mathbf{B}$ in Eq. (10d) cyclically. Using Eqs. (8a), (8d) and Eqs. (9b), (9c), Eq. (10d) can be rewritten as

$$\begin{aligned}-v\frac{\partial M_1^x}{\partial\xi}=&[M_0^y(\partial^2M_0^z/\partial\xi^2)-M_0^z(\partial^2M_0^y/\partial\xi^2)]\\&+2A/\mu_0\{M_0^yB_2^z-M_2^zB_0^y+M_2^yB_0^z-M_0^zB_2^y\}.\end{aligned} \tag{11}$$

To proceed further we represent the unperturbed uniform magnetization $\mathbf{M}_0$ in terms of polar coordinates. As the magnetization $\mathbf{M}_0$ is restricted to the y-z plane in the lowest order of perturbation [see Eq. (8d)], we choose ϕ as $\pi/2$ so that $\mathbf{M}_0$ takes the form

$$\mathbf{M}_0=(0,\sin\theta,\cos\theta). \tag{12}$$

Substituting Eqs. (8a), (8d), (9b), (9c), and (12) into the y and z component equations of $\mathbf{M}_1$ we obtain

$$-v\,\partial M_1^y/\partial\xi = 2A/\mu_0\{-M_1^xB_1^z+M_2^xB_0^z\}, \tag{13a}$$

$$-v\,\partial M_1^z/\partial\xi = 2A/\mu_0\{M_1^xB_1^y+M_2^xB_0^y\}. \tag{13b}$$

It is interesting to note that in Eqs. (11) and (13) the contribution due to Gilbert damping, i.e., the terms proportional to α, is absent. This is because the effect of damping is compensated by the nonlinearity. In the polar coordinate representation given in Eq. (12), Eq. (9d) takes the form

$$M_1^x=(\mu_0 v k/2AB_0^z)\,f, \tag{14}$$

where $f=\partial\theta/\partial\xi$. Using Eqs. (10b), (10c), (12), and (14) in Eq. (11) and differentiating with respect to ξ, after some lengthy algebra and calculations Eq. (11) can be made equivalent to the modified Korteweg–de Vries (MKDV) equation in the form

$$(\partial f/\partial\tau)+(3/2)\,\mu f^2\,(\partial f/\partial\xi)+\mu\,(\partial^3 f/\partial\xi^3)=0, \tag{15}$$

where $\mu=\mu_0^2 v k^3/8\epsilon_0 A^2$. The MKDV equation (15) is a completely integrable nonlinear evolution equation possessing N-soliton solutions [15]. For instance, the one-soliton solution of Eq. (15) can be written as

$$f=2a\,\mathrm{sech}\,a\zeta, \tag{16}$$

where $\zeta=\xi-\eta\tau$, $a^2=\eta/\mu$, and $\eta=$ const. Knowing f, θ can be calculated and hence from Eqs. (12) and (14) we obtain the components of magnetization as

$$M_1^x=\frac{\mu_0 a v k}{A}\mathrm{sech}\,a\zeta, \tag{17a}$$

$$M_0^y=1-2\;\mathrm{sech}^2 a\zeta, \tag{17b}$$

$$M_0^z=2\tanh a\zeta\;\mathrm{sech}\,a\zeta. \tag{17c}$$

Using Eqs. (17) in Eqs. (8b) and (8c), we obtain $B_0^y=k^{-1}(1-2\;\mathrm{sech}^2 a\zeta)$ and $B_0^z=2k^{-1}\tanh a\zeta\,\mathrm{sech}\,a\zeta$. Knowing $\mathbf{B}$ and $\mathbf{M}$, the magnetic field $\mathbf{H}$ of the EMW can be obtained using relation (2). From the above we observe that the excitation of the magnetization, the magnetic induction, and the magnetic field are governed by soliton modes, even in the presence of damping of small magnitude, thus indicating the possibility of lossless propagation of the EMW in a ferromagnetic medium as found experimentally in the case of garnets (see, for example, Ref. [16]).

IV. DAMPING OF ELECTROMAGNETIC SOLITON

In order to understand the effect of damping when it is significant, we redefine the damping parameter α as $\alpha=\varepsilon\alpha$ instead of $\alpha=\varepsilon^2\alpha$ as done earlier. We then substitute Eqs. (5), (6a), and (7) and $\alpha=\varepsilon\alpha$ in the one-dimensional component form of Eqs. (3) and (4) as before and solve the resultant equations. We find that the damping does not change the results at the orders of ε^0 and ε^1 and hence we have the same equations as found in Eqs. (8) and (9). However, at $O(\varepsilon^2)$, we obtain Eqs. (10a)–(10d), with the terms proportional to α in Eq. (10d) and in the y- and z-component equations of $\mathbf{M}_1$ replaced by new terms. For example, in Eq. (10d) the terms proportional to α are replaced by the following new terms:

$$\alpha\{B_1^x(M_0^{x^2}-1)+2B_0^xM_0^xM_1^x+M_1^x(M_0^yB_0^y+M_0^zB_0^z)+M_0^x(M_0^yB_1^y+M_1^yB_0^y+M_0^zB_1^z+M_1^zB_0^z)\}. \tag{18}$$

On solving Eq. (18) as in Sec. III, after lengthy calculations we end up with a perturbed MKDV equation with a dissipation term $\alpha J(\partial^2 f/\partial\xi^2)$, which upon making the transformations $f'\rightarrow f/2$ and $\tau'\rightarrow(\mu/2)\tau$ becomes

$$\frac{\partial f'}{\partial\tau'}+6f'^2\frac{\partial f'}{\partial\xi}+2\frac{\partial^3 f'}{\partial\xi^3}=-2\frac{\alpha J}{\mu}\frac{\partial^2 f'}{\partial\xi^2}, \tag{19}$$

where $J=4[\mu A/\mu_0 v k]$.

The effect of the structural disturbance on the soliton evolution due to the presence of the term proportional to α in Eq. (19) can be analyzed by using a perturbation theory following [17]. The results show that a small structural difference will lead to a slow variation of the soliton parameters and distortion of the soliton shape. The one-soliton solution of Eq. (19) is of the form

$$f'=2g_0(\tau')[\mathrm{sech}(z)-W(z,\tau')], \tag{20}$$

where $z=2g_0(\tau')[\xi-\phi_0(\tau')]$ and the parameters $g_0(\tau')$ and $\phi_0(\tau')$ are found from the relations

$$dg_0/d\tau'=\tfrac{1}{2}\int_{-\infty}^{\infty}\left(\frac{R}{\cosh z}\right)dz,$$

and

$$d\phi_0/d\tau'=4g_0^2+(1/4g_0^2)\int_{-\infty}^{\infty}\left(\frac{Rz}{\cosh z}\right)dz.$$

.

The correction to the soliton $W(z,\tau')$ is determined from a cumbersome expression that has the asymptotic form

$$W=\frac{1}{32g_0^4}z^2\exp(-z)\int_{-\infty}^{\infty}R\frac{dz}{\cosh z},\quad z\rightarrow\infty \tag{21a}$$

$$W=\frac{1}{32g_0^4}2\sigma' z\int_{-\infty}^{\infty}R\;dz,\quad z\rightarrow-\infty, \tag{21b}$$

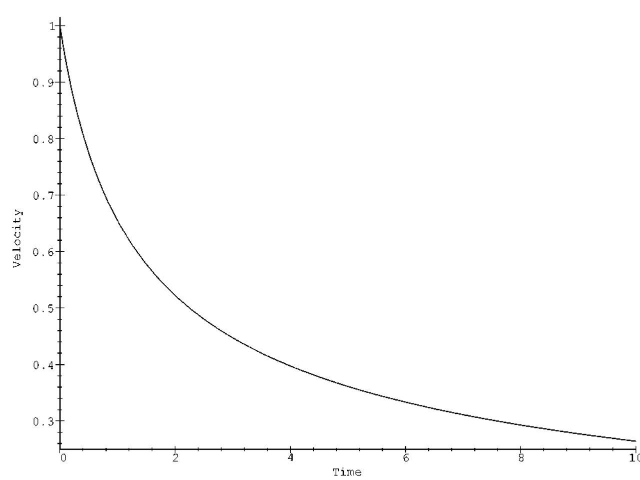

FIG. 1. Deceleration of the electromagnetic soliton in a dissipative ferromagnetic medium [Eq. (21) for $\alpha=1.0$, $J/\mu=-1/4$, and $u(0)=1$].

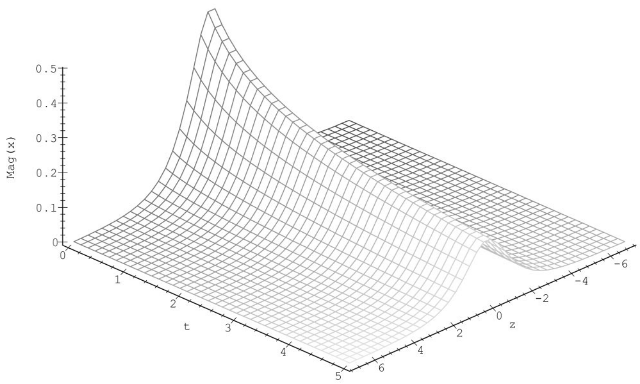

FIG. 2. Damping of the magnetization soliton (M_1^x) in a dissipative ferromagnetic medium.

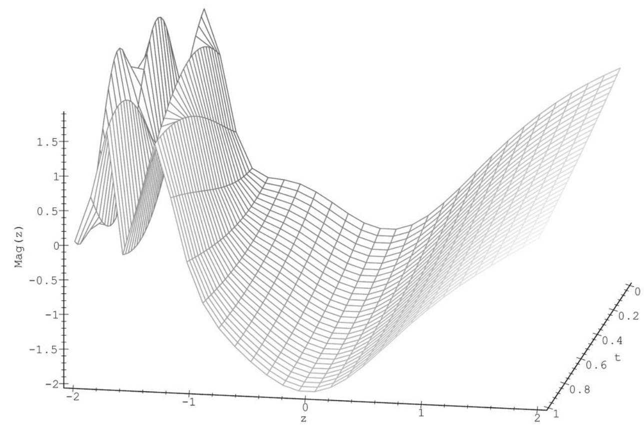

FIG. 3. Damping of the magnetization soliton (M_0^z) in a dissipative ferromagnetic medium.

where $1/\sigma' = 8\int g_0^2 d\tau'$. Here R stands for the right-hand side of Eq. (19). On evaluating the integrals in the g_0 and ϕ_0 equations the velocity $u(\tau') = dg_0/d\tau'$ of the electromagnetic soliton is found to evolve according to

$$u(\tau') = u(0)\{1 + (4/3)[-\alpha J/\mu]u^2(0)\tau'\}^{-1/2}. \quad (22)$$

From Eq. (22) we observe that the effect of Gilbert damping leads to the deceleration of the electromagnetic soliton, which is illustrated in Fig. 1. In accordance with Eq. (20), the localization region of the soliton widens. From Eq. (21) the variation of the soliton shape is given by the asymptotic relations $2g_0W = (-\alpha J/6\mu)u(\tau')z^2\exp(-z)$ as $z \to \infty$ and $2g_0W = 0$ as $z \to -\infty$. Knowing f' from Eq. (20), we can calculate θ from the relation $f' = \frac{1}{2}(\partial\theta/\partial\xi)$ and hence the magnetization of the medium using Eqs. (12) and (14) and consequently the magnetic induction and the magnetic-field component of the EMW can also be computed. In Fig. 2 we illustrate how the x component of the magnetization $\mathbf{M}_1$ [Mag(x)] gets damped. It is observed that the perturbation W distorts the shape of the soliton only at one end of it, thus displaying a certain stability with respect to structural excitation. This is illustrated in Fig. 3 in the case of the z component of the magnetization $\mathbf{M}_0$ [Mag(z)]. The above conclusions of the soliton damping and distortion will also hold true for the magnetic induction and magnetic field of the EMW.

V. CONCLUSIONS

In this paper we investigated the nature of propagation of the EMW in an untreated isotropic ferromagnetic medium with Gilbert damping and free charges. We found that when the magnitude of damping is small the results are similar to a charge-free ferromagnetic medium without damping and therefore the excitations of the magnetization, the magnetic induction, and hence the magnetic field of the EMW are governed by solitons. When the magnitude of damping increases, the above electromagnetic solitons undergo the following changes. As time increases the electromagnetic solitons decelerate and the amplitude of the solitons decreases and gets damped. Further, the dissipation due to Gilbert damping of the medium introduces a slight distortion in the shape of the electromagnetic solitons only at one end, thus showing a certain structural stability.

ACKNOWLEDGMENTS

The work of M.D. forms part of a major DST project. V.V. acknowledges R. Amuda and K. Manivannan for discussions.

[1] J. L. Simonds, Phys. Today **48**(4), 26 (1995).
[2] M. Daniel, V. Veerakumar, and R. Amuda, Phys. Rev. E **55**, 3619 (1997).
[3] R.F. Soohoo, *Theory and Applications of Ferrites* (Prentice-Hall, London, 1960).
[4] F. G. Bass and N. N. Nasonov, Phys. Rep. **189**, 165 (1990), and references therein.
[5] I. Nakata, J. Phys. Soc. Jpn. **60**, 2179 (1991).
[6] H. Lablond, J. Phys. A **29**, 4623 (1996).
[7] J. D. Jackson, *Classical Electrodynamics* (Wiley Eastern, New York, 1993).
[8] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).
[9] H. B. Callen, *Relaxation and Resonance in Magnetic Systems* (Oliver and Boyd, Edinburgh, 1962).
[10] M. Lakshmanan, Phys. Lett. **61A**, 53 (1977).
[11] M. Lakshmanan and K. Nakamura, Phys. Rev. Lett. **53**, 2497 (1986).
[12] M. Lakshmanan and M. Daniel, Phys. Rev. B **24**, 6751 (1981).
[13] T. Kakutani and H. Ono, J. Phys. Soc. Jpn. **26**, 1305 (1969).
[14] I. Nakata, J. Phys. Soc. Jpn. **60**, 77 (1991).
[15] M. Wadati, J. Phys. Soc. Jpn. **32**, 1681 (1972).
[16] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. Lett. **70**, 1707 (1993).
[17] V. M. Karpman and E. M. Maslov, Zh. Eksp. Teor. Fiz. **73**, 538 (1977).